# Multi-objective control of active distribution systems incorporating various types of distributed energy resources

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***Abstract*: This paper deals with the management of operational constraints in active distribution systems hosting different types of distributed energy resources (DERs). The core of the proposed methodology is a multi-period optimal power flow (OPF) model that includes several optimization objectives, in order to adapt to the particular needs of the distribution system at different times. The model also integrates the control of certain distribution system components, such as remotely controlled switches on shunt capacitors, and produces as output the signals that should be used to regulate the operation of the available DERs. The performance of the method is demonstrated by comparing the results produced for each of the different optimization objectives for identical system input, indicating that the proposed model can effectively function under a wide range of external conditions.**

## NOMENCLATURE

*Sets*

| | |
|---|---|
| $\Omega_N$ | Set of system buses |
| $\Omega_{N,i}$ | Set of system buses connected to bus $i$ |
| $\Omega_L$ | Set of distribution lines |
| $\Omega_{DG}$ | Set of buses with distributed generation |
| $\Omega_{ST}$ | Set of buses with storage (ST) units |
| $\Omega_{CHP}$ | Set of buses with Combined Heat and Power (CHP) units |
| $\Omega_C$ | Set of buses with shunt capacitor banks |

*Parameters*

| | |
|---|---|
| $V^{nom}$ | Nominal voltage of the system |
| $r_{ij}$ , $x_{ij}$ | Resistance / reactance of line $i-j$ |
| $b_{ij}$ , $g_{ij}$ | Susceptance / conductance of line $i-j$ |
| $P_{d,i}$ , $Q_{d,i}$ | Active / reactive load demand at bus $i$ |
| $P_{DG,i}^{min}$ , $P_{DG,i}^{max}$ | Lower and upper limit of the active power injected by the Distributed Generation (DG) unit at bus $i$ |
| $c_{DG,i}$ | Price for the remuneration of active energy injected from the DG unit at bus $i$ |
| $\eta_{ST,i}^{C}$ , $\eta_{ST,i}^{D}$ | Charging / discharging efficiency of the storage unit at bus $i$ |
| $c_{ST,i}^{C}$ , $c_{ST,i}^{D}$ | Price of energy charging / discharging for the storage unit at bus $i$ |
| $E_{ST,i}^{C,lim}$, $E_{ST,i}^{D,lim}$ | Charging / discharging energy storage limit of the storage unit at bus $i$ (positive values) |
| $E_{ST,i}^{0}$ , $E_{ST,i}^{F}$ | Initial / final energy stored in the storage unit at bus $i$ |
| $P_{CHP,i}^{min}$ , $P_{CHP,i}^{max}$ | Lower and upper limit of the active power produced by the CHP unit at bus $i$ |
| $\alpha_{CHP,i}$ , $\beta_{CHP,i}$ , $\gamma_{CHP,i}$ | Generation cost coefficients of the CHP unit at bus $i$ |

*Variables*

| | |
|---|---|
| $V_i$ , $\theta_i$ | Voltage magnitude and angle of bus $i$ |
| $\theta_{ij}$ | Voltage angle difference between buses $i$ and $j$ |
| $I_{ij}$ | Current flowing through line $i-j$ |
| $P_i$ , $Q_i$ | Net active / reactive power injection at bus $i$ |
| $Q_{C,i}$ | Reactive power injected by the capacitor bank at bus $i$ |
| $P_{DG,i}$ , $Q_{DG,i}$ | Active / reactive power injected by the DG unit at bus $i$ |
| $E_{ST,i}^{C}$ , $E_{ST,i}^{D}$ | Charging / discharging energy stored to / drawn from the storage unit at bus $i$ |
| $P_{CHP,i}$ | Active power injected by the CHP unit at bus $i$ |

## I. INTRODUCTION

Power distribution systems have been recently undergoing a gradual evolution in concept and form, by replacing, to some extent, inflexible loads and conventional power plants with a large number of distributed generators, energy storage units and flexible loads, all of which fall under the term distributed energy resources (DERs) [1]. Among other advantages, DERs have the potential to provide various services to the distribution network, such as network and generation capacity, ancillary services like voltage and frequency regulation, reliability and resiliency [2].

However, in order to harness the benefits of the integration of DERs in distribution networks, the implementation of suitable control and coordination strategies is needed. In this regard, the control strategies can be centralized ([3],[4]), where signals that regulate the operation of the network components are issued by a single control center, or decentralized ([5],[6]), where there exist multiple local agents that coordinate system components.

In this paper, a centralized control strategy is proposed in the form of an OPF problem to provide distribution system operators (DSOs) with a flexible tool to efficiently manage a variety of different types of DERs, like distributed generation units, energy storage units as well as combined heat and power (CHP) units. Due to the non-homogeneity of different regulatory frameworks and contractual energy agreements among DSOs, the proposed model aims to offer a core control strategy that can be fine-tuned by the DSO to suit his individual network case.

## II. Mathematical Formulation

The proposed methodology uses the OPF problem as its basis. The control variables of the problem are the dispatch levels of the various DERs, the current flows in the grid, the voltage angle and magnitude for each bus as well as the connection status of the switchgear in shunt capacitors.

Since the OPF problem is a steady-state analysis, it is common practice to split the desired simulation horizon into time intervals, usually hourly ones. Thus, it should be noted that the presented formulation contains the parameters and variables involved in one hourly dispatch period. For a T-hour period, each of the equations (1)-(9) is formulated for each hour, including parameters and variables of the same time indicator $t \in \{1,2,..,T\}$. The problem is then solved collectively for the whole T-hour period.

One of the major consequences of this is that parameters and variables that represent energy produced / consumed within an hourly dispatch period also describe the mean power produced / consumed during the same period. This allows variables expressed in power units to be used in cost equations that use cost parameters expressed per unit of energy.

### A. *Distribution System Constraints*

In the basic OPF problem, the equality constraints include the power flow equations and the power balance for each bus. The inequality constraints include the upper and lower limits of the control variables along with the thermal limits of the lines.

The standard power flow equations for the net active and reactive power injection at bus $i$, $P_i$ and $Q_i$, respectively, are the following:

$$P_i = \sum_{j \in \Omega_{N,i}} [\, V_i^2 \, g_{ij} - |V_i| \, |V_j| \left( g_{ij} \cos\theta_{ij} + b_{ij} \sin\theta_{ij} \right) ] \quad (1)$$

$$Q_i = \sum_{j \in \Omega_{N,i}} [\, -V_i^2 \, b_{ij} - |V_i| \, |V_j| \left( g_{ij} \sin\theta_{ij} - b_{ij} \cos\theta_{ij} \right) ] \quad (2)$$

The same net power injections, as expressed by the power injected/demanded by network components for each bus $i$, are:

$$P_i = P_{DG,i} + P_{CHP,i} + E_{ST,i}^{d} - E_{ST,i}^{c} - P_{d,i} \quad (3)$$

$$Q_i = Q_{DG,i} + Q_{C,i} - Q_{d,i} \quad (4)$$

Further operational constraints for the system variables include voltage limits for all $i \in \Omega_N$ and current limits for all $(i,j) \in \Omega_L$. Shunt capacitor banks can be modeled as switching components. Those provide the network with reactive power in discrete increments when necessary, ranging from zero to their full capacity. However, their presence inserts discrete variables into the model, making it harder to solve.

### B. *Distributed Generation Units*

The types of DG units connected to the network may vary, with the most prominent ones being wind or photovoltaic (PV) parks but other types, such as small hydropower units, are not uncommon. Regardless of their nature, limits for the operation of those DG units are to be set by the DSO in order to achieve optimal distribution system management for each dispatch period. The major relevant constraint, for all $i \in \Omega_{DG}$, is:

$$P_{DG,i}^{min} \leq P_{DG,i} \leq P_{DG,i}^{max} \quad (5)$$

### C. *Storage Units*

Storage units have specific charging / discharging limits, as per the following constraints, for all $i \in \Omega_{ST}$ :

$$E_{ST,i}^{C} \leq E_{ST,i}^{C,lim} \quad (6)$$

$$E_{ST,i}^{D} \leq E_{ST,i}^{D,lim} \quad (7)$$

Subsequently, the total charging / discharging of the storage units during one dispatch period should follow the imposed initial and final energy storage values. Thus, for all $i \in \Omega_{ST}$ [7]:

$$E_{ST,i}^{F} - E_{ST,i}^{0} = E_{ST,i}^{C} \, \eta_{ST,i}^{C} - \frac{E_{ST,i}^{D}}{\eta_{ST,i}^{D}} \quad (8)$$

### D. *CHP Units*

The constraints for the modeling of the CHP units during one dispatch period are essentially the same as those of a conventional generator, namely the upper and lower active power production limits. For all $i \in \Omega_{CHP}$ :

$$P_{CHP,i}^{min} \leq P_{CHP,i} \leq P_{CHP,i}^{max} \quad (9)$$

CHP units are also constrained by the presence of ramp rate limits. Those cause the active power production of a CHP unit during a particular dispatch period to be dependent on its active power production during the exact previous dispatch period, limiting the degrees of freedom in the model.

### E. *Objective Function*

The solution of the OPF problem by the DSO can have varying objectives. In the current formulation, three objectives have been selected for examination: the cost of power generation, the distribution system losses and the voltage deviation from the nominal value.

In order to determine the total cost of power generation and transactions, we take into account DG units, energy storage units as well as CHP units. The respective total costs for each unit type (during one hourly dispatch period) are as follows:

$$\sum_{i \in \Omega_{DG}} c_{DG,i} \, P_{DG,i} \quad (10.a)$$

$$\sum_{i\in\Omega_{ST}} (-c_{ST,i}^{C}\, E_{ST,i}^{C} + c_{ST,i}^{D}\, E_{ST,i}^{D}) \tag{10.b}$$

$$\sum_{i\in\Omega_{CHP}} (\, \alpha_{CHP,i}\, P_{CHP,i}^{2} + \beta_{CHP,i}\, P_{CHP,i} \, + \gamma_{CHP,i}) \tag{10.c}$$

The distribution system losses are given by:

$$\sum_{(i,j)\in\Omega_L} \frac{I_{ij}^2}{g_{ij}} \tag{11}$$

Finally, the bus voltage deviation from the nominal value is:

$$\sum_{i\in\Omega_N} (V_i - V^{nom})^2 \tag{12}$$

While each of the three presented objectives in (10)–(12) is equally viable as a core of the objective function of the model, a combined approach can also be considered. By assigning adjustable weighting coefficients to each, it is possible to combine the previous expressions. The resulting problem is a Mixed Integer Non Linear Problem (MINLP) where the general objective function consists of the weighted sum of the expressions in (10)-(12) for all hourly dispatch periods.

## III. Case Studies and Results

The proposed mathematical model has been developed in GAMS using the BONMIN solver and was tested on a 15-bus, 20 kV, power distribution system. The parameters of the system are provided in Table 1. Data of connected DERs is provided in Table 2.

The network has been tested during a three hour period $T = \{t_1, t_2, t_3\}$ . For the same data input, four different optimization modes (strategies) of the model have been considered. The first three assume general objective functions identical to each one of the three partial objectives presented in Section II. For the fourth strategy a combined approach has been selected, using the complete form of the function with multiple objectives. Finally, the following assumptions have been made:

a) The load reactive power is assumed to be 20% of the load active power at all times.
b) The storage units at buses 9 and 11 begin fully charged at their maximum capacity of 0.5 MWh and 0.8 MWh, respectively. The energy storage values at the end of the third hour are required to be: $E_{ST,9}^{F}$ = 0.3 MWh and $E_{ST,11}^{F}$ = 0.7 MWh.
c) Two capacitor banks, each of total reactive power of 400 kVar injected in 100 kVar increments (5 states), are assumed connected at buses 4 and 11, respectively.

The impact of each strategy on each of the partial objectives after the whole simulation period is displayed in Table 3, next to an initial network state where no DERs are connected. It is evident that modes M1–M3 are most effective in minimizing their respective objective, while M4 is more of a "middle ground" approach. The energy injections of the DER units for each strategy are presented in Table 4.

### A. *Strategy S1– Cost Minimization*

In this strategy, the objective function is defined as the sum of expressions appearing in (10.a), (10.b) and (10.c) for all three hours. No weighting coefficients are assigned, such that all DER units are equally eligible for cost reduction.

The simulation results show that each DER contributes in the cost minimization as follows:

a) The PV units drop their active power production to their lowest possible values allowed by the constraints.
b) The CHP unit does not inject any active power to the network. However, it still contributes to the overall cost due to the presence of the constant $\gamma_{CHP,i}$ in the objective function.
c) The two storage units drop to energy levels lower than the final desired ones at the end of the second hour and charge the maximum allowed amount during the third hour. Indeed, it is economically advantageous for the DSO to draw as much energy as possible from the storage units before charging them back to the constraint defined values, since the charging cost is higher than the discharging one throughout the whole time period.

### B. *Strategy S2 – System Losses Minimization*

The consequences of considering system losses as the objective are as follows:

a) The PV unit at bus 7 operates at its maximum power throughout the whole period. The higher capacity unit at bus 5 though adjusts its active power injection following load demand, operating at its maximum power during the first hour and gradually reducing its production.
b) The CHP unit also reaches its upper production limit, maintaining it during all three hours.
c) The storage units at buses 9 and 11 display simpler behavior as in M1, essentially seeking to reach the imposed energy charge levels as directly as possible, which they do at the second (for bus 9) and first (for bus 11) hour of the simulation, respectively.

Since this mode focuses on mitigating grid losses, it is reasonable to see DERs be used to their full extent, by directly supplying loads in neighboring buses and thus reducing the active power flows through the system lines.

TABLE 1
Network Parameters

| **Bus $i$** | **Bus $j$** | $r_{ij}$ **(Ω)** | $x_{ij}$ **(Ω)** | $P_{D,j}(t_1)$ **(MW)** | $P_{D,j}(t_2)$ **(MW)** | $P_{D,j}(t_3)$ **(MW)** |
|---|---|---|---|---|---|---|
| 1 | 2 | 0.0922 | 0.0470 | 0.6 | 0.5 | 0.2 |
| 2 | 3 | 0.4930 | 0.2511 | 0.5 | 0.3 | 0.1 |
| 3 | 4 | 0.3660 | 0.1864 | 0.7 | 0.3 | 0.2 |
| 4 | 5 | 0.3811 | 0.1941 | 0.2 | 0.2 | 0.3 |
| 5 | 6 | 0.8190 | 0.7070 | 0.2 | 0.2 | 0.1 |
| 2 | 7 | 0.1640 | 0.1565 | 0.3 | 0.1 | 0.4 |
| 7 | 8 | 1.5042 | 1.3554 | 0.4 | 0.6 | 0.1 |
| 8 | 9 | 0.4095 | 0.4784 | 0.5 | 0.4 | 0.3 |
| 9 | 10 | 0.7089 | 0.9373 | 0.8 | 0.5 | 0.2 |
| 3 | 11 | 0.4512 | 0.3083 | 0.6 | 0.1 | 0.1 |
| 11 | 12 | 0.8980 | 0.7091 | 1.2 | 0.8 | 0.5 |

TABLE 2
DATA OF DER UNITS IN THE NETWORK

| DERs | Bus | Component Parameters | |
|---|---|---|---|
| DG Units (PV) | 5 | $P_{DG,i}^{min} = 0.4\ (MW),\ P_{DG,i}^{max} = 1(MW),$ $c_{DG,i} = 80(€/MWh)$ | |
| | 7 | $P_{DG,i}^{min} = 0\ (MW),\ P_{DG,i}^{max} = 0.6(MW),$ $c_{DG,i} = 88(€/MWh)$ | |
| Storage Units | 9 | $c_{ST,i}^{C} = 92(€/MWh)$ | $\eta_{ST,i}^{C} = 0.78,\ \eta_{ST,i}^{D} = 0.72$ $(E_{ST,i}^{C,lim}, E_{ST,i}^{D,lim}) = (0.1, 0.15)(MWh)$ |
| | 11 | $c_{ST,i}^{D} = 85(€/MWh)$ | $\eta_{ST,i}^{C} = 0.85,\ \eta_{ST,i}^{D} = 0.8$ $(E_{ST,i}^{C,lim}, E_{ST,i}^{D,lim}) = (0.4, 0.2)(MWh)$ |
| CHP Units | 3 | $P_{CHP,i}^{min} = 0(MW),\ P_{CHP,i}^{max} = 0.8(MW)$ $\alpha_{CHP,i} = 0.00278\ (€/MW^2)$ $\beta_{CHP,i} = 6.56(€/MW),\ \gamma_{CHP,i} = 223(€),$ | |

### C. Strategy S3 – Voltage Deviation Minimization

Deploying the DERs to minimize the bus voltage deviations produces the following results:

a) The PV unit at bus 7 keeps operating at its maximum power, while the PV unit at bus 5 keeps its active power production at its minimum.
b) The produced active power of the CHP unit seems to be fluctuating at about half its maximum limit, though without displaying any obvious inclination to follow load demand.
c) The storage units follow exactly the same operation patterns as in M2. As far as cost is not involved, there seems to be no incentive to diverge from directly satisfying the final energy level constraints.

### D. Strategy S4 – Combined Objective Function

In this strategy, the objective function is the sum of all the terms in (10)–(12). The obtained results are:

a) The PV unit at bus 7 operates constantly at its maximum power, while the PV unit at bus 5 reaches its maximum power at the first hour while producing its minimum power during the next two hours.
b) The CHP unit injects its maximum possible power into the system during the first two hours, while slightly decreasing its output during the third hour.
c) The storage units seem to follow the same discharging course as in M2 and M3. A heavier focus on cost minimization is probably needed to achieve the behavior displayed in M1.

## IV. CONCLUSION

The proposed mathematical model is tested on a power distribution system during a three hour period of varying load demand. By applying different objectives on the basic model, a wide range of output and DER behavior is achieved, favoring different system variables each time.

The presented solving procedure can be considered a comprehensive approach, achieving optimal results for the period of interest as a whole. However, this comes at the cost of increased computational time. In that regard, certain "greedy" methods, which solve each hourly sub-problem individually, fare better, as those sub-problems are considerably easier than the general one.

Proposals for future development of the method include the integration of network reconfiguration schemes, the inclusion of additional DER types (such as flexible resource units) as well as the definition of an optimal weight tuning method for the model to self-adapt to specific network and market conditions.

TABLE 3
COLLECTIVE RESULTS OF ALL STRATEGIES

| | No DER | S1 | S2 | S3 | S4 |
|---|---|---|---|---|---|
| Generation Cost (€) | - | 470.92 | 755.88 | 667.04 | 701.97 |
| Grid Losses (MWh) | 0.0784 | 0.0641 | 0.0350 | 0.0519 | 0.0442 |
| Avg. Volt. Dv. (pu) | 0.0029 | 0.0028 | 0.0974 | 0.0022 | 0.0027 |

TABLE 4
DER ENERGY INJECTIONS (MWH) PER DER FOR THE 3-HOUR PERIOD

| Strategy | Hour | PV at bus 5 | PV at bus 7 | CHP at bus 3 | ST at bus 9 | ST at bus 11 |
|---|---|---|---|---|---|---|
| S1 | t1 | 0.4000 | 0.0000 | 0.0000 | 0.1304 | 0.1651 |
| | t2 | 0.4000 | 0.0000 | 0.0000 | 0.1417 | 0.1869 |
| | t3 | 0.4000 | 0.0000 | 0.0000 | -0.1000 | -0.4000 |
| S2 | t1 | 1.0000 | 0.6000 | 0.8000 | 0.1500 | 0.0800 |
| | t2 | 0.8907 | 0.6000 | 0.8000 | 0.0060 | 0.0000 |
| | t3 | 0.5406 | 0.6000 | 0.7800 | 0.0000 | 0.0000 |
| S3 | t1 | 0.6143 | 0.6000 | 0.4473 | 0.1400 | 0.0800 |
| | t2 | 0.4000 | 0.6000 | 0.3473 | 0.0761 | 0.0000 |
| | t3 | 0.4000 | 0.6000 | 0.4657 | 0.0000 | 0.0000 |
| S4 | t1 | 0.9650 | 0.6000 | 0.8000 | 0.1500 | 0.0800 |
| | t2 | 0.4000 | 0.6000 | 0.8000 | 0.0660 | 0.0000 |
| | t3 | 0.4000 | 0.6000 | 0.7061 | 0.0000 | 0.0000 |

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